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ABSTRACT

Curricular analyses, written by experienced teachers and experts, of the 1978 Illinois Inventory of Educational Progress (IIEP) results, and teacher expectations regarding mathematics are presented. A random sample of 7,200 Illinois students within 360 schools (grades 4, 8, 11) were tested; results reflected their educational performance on the following topics: problem solving; applying mathematics to everyday situations; alertness to the reasonableness of results; estimation and approximation; computational skills; geometry; measurement; reading, interpreting, and constructing tables, charts, and graphs; and using mathematics to predict. General conclusions emerging from the curricular analyses included: (1) students had good understanding of the basic properties of numbers and scored high in addition and multiplication, but scored low in subtraction and division; (2) fractions were well taught initially, but not retained well; (3) student success depended on how relevant items were in their daily lives; (4) scores in basic metric units improved considerably from 1976 to 1978. Mathematics teachers in sampled schools at all grade levels consistently overestimated actual student performance, their expectations becoming more discrepant on objectives involving computation, understanding, and problem solving. (RL)

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ED191889

ILLINOIS INVENTORY OF EDUCATIONAL PROGRESS

Mathematics Curricular Analyses and Teacher Expectation Results

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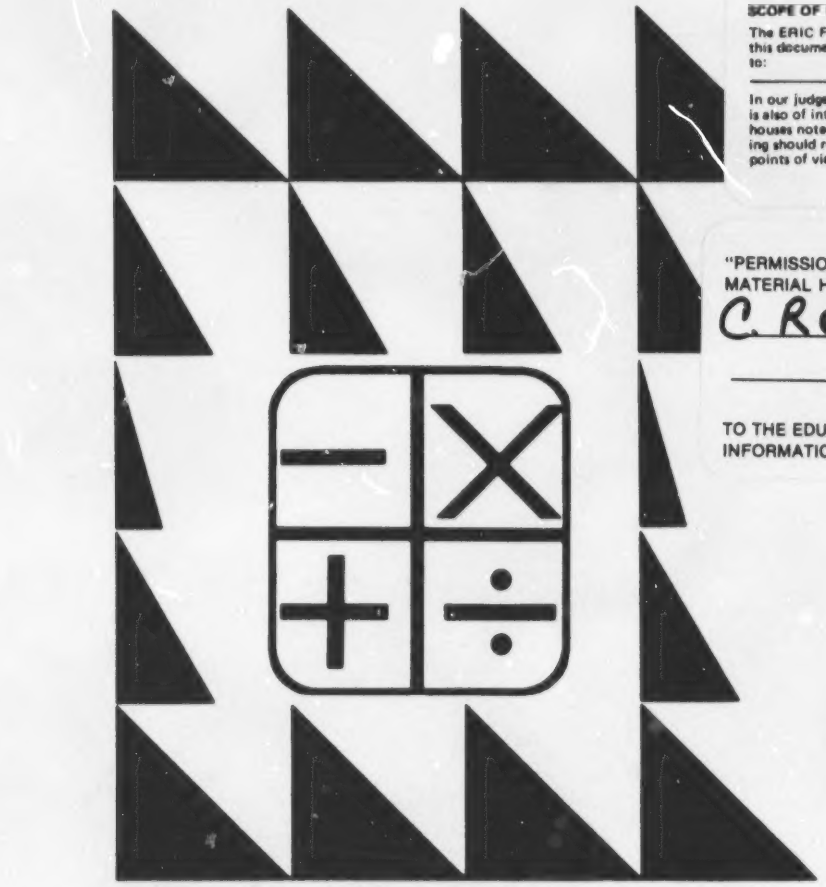

IIEP 1978

DONALD F. MUIRHEID, CHAIRMAN
ILLINOIS STATE BOARD OF EDUCATION

Prepared By:
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FOREWORD

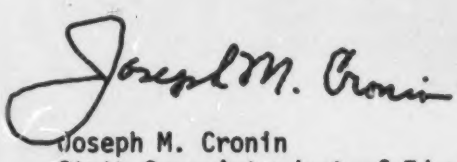
The 1978 Illinois Inventory of Educational Progress (IIEP) was a compilation of tests given to fourth, eighth, and eleventh grade students in mathematics, reading, and citizenship.

The present report contains curricular analyses of the mathematics results and the teacher expectation results. The Illinois State Board of Education has published a number of reports on the 1978 IIEP results.

The teacher expectation survey asked teachers within the three subject areas to give their expectations of student performance on the test items; the teacher expectation survey was conducted at the same time the test was given.

It is hoped that this report will be helpful to all Illinois educators and citizens concerned about the educational accomplishments of our schools.

This report was prepared by C. Thomas Kerins, Mervin M. Brennan, Kenneth R. Redding, Leslie J. Fyans, Jr., and Mary E. Raikes. Their efforts and the efforts of many Illinois educators are gratefully acknowledged.



Joseph M. Cronin
State Superintendent of Education

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INTRODUCTION

Each spring the Illinois State Board of Education administers the Illinois Inventory of Educational Progress (IIEP).

Purpose and Goals

The purpose of the IIEP is to measure what Illinois students know and can do in certain subject matter areas and to provide that information to educational decision makers.

The goals of the IIEP are:

1) to make available reliable, valid, and relevant data on the educational achievement of Illinois students, and

2) to identify any trends (growth, stability, or decline) in educational achievement which occur over time.

Results of the 1978 IIEP

The present report contains curricular analyses of the 1978 IIEP results and teacher expectations regarding mathematics. Experienced teachers and experts wrote the curricular analyses. The analyses are solely those of the experts and are not to be taken as the official position of the State Superintendent of Education or the Illinois State Board of Education.

At the time of the IIEP administration to students, teachers in participating schools were asked to critique each test item. The teachers reported whether or not students had been exposed to the subject matter and whether or not the item was of appropriate difficulty level. They estimated the percentage of students who would obtain the correct answer.

The processes used to obtain the curricular analyses and teacher expectations are described in Appendices A and B.

Student Sampling Model

A random sample with two sampling stages was used to select those students attending Illinois public schools who participated. In the first stage, a random sample of approximately 360 schools throughout the state was drawn. Then a random sample of the students within these schools was drawn from the lists of eligible students submitted by the school. The final sample included about 7,200 Illinois students at the fourth, eighth, and eleventh grades. The sample reflected the educational performance of students approaching the end of primary, elementary, and secondary levels of education.

IIEP Reports

Appendix E is a list of publications regarding the IIEP results.

SECTION I

CURRICULAR ANALYSES BY EXTERNAL REVIEWERS

CHAPTER 1

OVERVIEW ANALYSIS OF THE MATHEMATICS RESULTS

Introduction

The mathematics portion of the IIEP measured many of the general mathematics topics which are taught to students in their elementary, junior high, and high school mathematics courses. The IIEP mathematics objectives are presented in Appendix C. All major categories were sampled. Some categories, such as numbers and consumer mathematics were sampled heavily, while other areas such as geometry and statistics received less attention.

The IIEP has taken into account the work of various professional organizations. In 1977, the National Council of Supervisors of Mathematics issued a position paper on Basic Mathematical Skills. The paper listed the following vital areas: problem solving; applying mathematics to everyday situations; alertness to the reasonableness of results; estimation and approximation; appropriate computational skills; geometry; measurement; reading, interpreting, and constructing tables, charts, and graphs; using mathematics to predict; and computer literacy. The IIEP tested in all these areas except computer literacy, and computer literacy should probably not be tested by a paper and pencil test. Analysis of the three tests indicate that they cover a wide variety of areas. The mathematics items are typical of test items and instructional content everywhere in the United States.

Several general conclusions emerged from the curricular analyses and are presented in the form of headlines.

- STUDENTS INDICATED GOOD UNDERSTANDING OF THE BASIC PROPERTIES OF NUMBERS.

- SCORES IN BASIC METRIC UNITS IMPROVED CONSIDERABLY FROM 1976 TO 1978.

- STUDENTS SCORED HIGH IN ADDITION AND MULTIPLICATION.

- STUDENT SUCCESS DEPENDED ON HOW RELEVANT ITEMS WERE IN THEIR DAILY LIVES.

- FRACTIONS SEEMED TO BE WELL TAUGHT INITIALLY, BUT NOT RETAINED WELL IN SUBSEQUENT YEARS.

- STUDENT ACHIEVEMENT WAS HIGH ON PROBLEMS REQUIRING ONE STEP. WHEN MORE STEPS WERE NEEDED TO SOLVE A PROBLEM, STUDENT ACHIEVEMENT WAS LOWER.

- STUDENTS WERE WEAK IN SUBTRACTION AND DIVISION.

Summary of Results

The results of the IIEP give us considerable information. They indicate that students were strong in their understanding of the basic properties of numbers and in their learning of the recently introduced area of metrics. Results were high for items of addition and multiplication and on items whose content was familiar. Although few students take courses in algebra, nearly half the students attained the correct answers on the algebra items. Students did similarly well on items concerning probability and statistics even though these areas are not emphasized in most general mathematics courses. Students were weak in their understanding of advanced concepts, such as exponents, did poorly at all grade levels on subtraction items, were unable to add fractions with different denominators correctly, and did not apply the principles of ratios and proportions with great success.

Objectives 1 - 4: Numeration Concepts

Students were strong in some concepts and weak in others. For example, fourth grade students showed a good grasp of whole numbers and eighth grade students had difficulty with exponents. Only 47% of the eighth grade students recognized 4^3, but 71% of them responded correctly to the item $9\times9\times9\times9$. The $9\times9\times9\times9$ is a harder multiplication problem than $4\times4\times4$. Thus, there seems to be a language development problem. Since students can compute $9\times9\times9\times9$, they can probably compute $4\times4\times4$. However, they do not appear to understand the notation 4^3.

A language development deficiency was also indicated by several items related to consumer mathematics and by the variation in the percentages correct for the ratio and proportion items. The students probably had not used ratios and proportions in attempting to solve the problems. If they had, the percentages correct would have been more consistent across items.

Objectives 5 - 8: Number Properties

The items on number properties produced acceptable results. The students seemed to understand comparing numbers, the identification of digits, and renaming from an expanded form to the standard form. The items in this category would have given us more information if they had focused on mathematics knowledge which is more generalizable. Thus, expanded numbers should have been directly related to the kind of renaming that students do in order to perform operations, such as in renaming one "10" as ten "1's" or one "100" as ten "10's." In that way, we could test material of one level which is usable at another level and be able to determine whether instructional programs are teaching students the key elements necessary for success.

Objectives 9 - 36: Numbers

Most students seemed to understand comparisons of numbers, identification of digits, and renaming from an expanded form to the standard form. Since they understood the concepts, one would have expected them to do better for operations with numbers than they did. Perhaps the theory needs to be related more explicitly to application and repeated constantly, because the student scores are reasonable but not as high as one would hope.

Objectives 9 - 12: Whole Numbers

In regard to whole numbers at the fourth grade level, the results seem to be basically what one would expect. For the items, 67% multiplied correctly, while 50% divided successfully. The scores in subtraction were low. Low scores occurred at all three grade levels on subtraction items, and the issue of subtraction should be looked at very carefully by mathematics educators.

Objectives 13 - 16: Fractions

It was impressive that 71% of the eighth and eleventh grade students multiplied the fraction item correctly. Perhaps this was because the multiplication process is straightforward and uncomplicated. In contrast, only 30% of the eighth graders and 47% of the eleventh graders added one-half plus one-third correctly. The low scores for addition of fractions are reason for concern.

Concern about fractions was also dramatized by the differences between fourth and eighth grade results. It was particularly disturbing that on a pictorial display of a circle with shaded areas to represent fractional parts, fourth graders did better than eighth graders. Fourth graders evidently see fractional parts pictorially and eighth graders do not. It would seem that initial instruction in fractions at the fourth grade level is quite successful. The question is whether the follow through in the intermediate and upper grade levels needs to be revised, since it does not appear to be as successful.

Currently there are suggestions for a possible reduction in the emphasis on fractions at all levels as a result of the metric system and the calculator. However, the present results may indicate that fractions deserve more, rather than less, attention.

Objectives 37 - 40: Measurement

The percentages correct on items regarding the customary and metric units of measurement were similar. Since students have more experience in and out-of-school with the customary units, the results are encouraging for metrics. Students did quite well on the metric items. The fact that 84% of the eleventh graders recognized liters as a volume measure similar to gallons, 82% realized that kilograms are a measure of weight, and 51% knew the number of centimeters in a meter is good, given the experience students usually have with the metric system. Knowing that a square centimeter is about the size of a thumbnail is sophisticated. Students have had the opportunity to relate a square centimeter to their thumbnail. It is not at all certain that they would be that sure of the size of a square inch. Where students had problems with metrics, it could be attributed to things other than measurement, such as problem solving. Proponents of metrics have argued that the metric system is simple and that it can be applied to all appropriate problems successfully once it is learned. Future tests may be able to test that hypothesis.

The emphasis on metrics has allowed us to start from scratch, take a whole new look at measurement, and develop new strategies for teaching it. It has also provided an opportunity to retrain teachers in how to teach measurement. Because metrics is seen as a whole new area, teachers have approached it with an open mind, an admission of not knowing, and a willingness to be taught how to teach it to students. In the past, teachers frequently assumed that students understood volume, bypassed any exposures to experience and started with abstract symbols V=LWH (volume equals length times width times height). But teachers will not teach volume in metric units without having students build something that has volume. The results of the test seem to indicate that we can teach measurements better by emphasizing student experience. Students seemed to have more trouble with concepts surrounding measurement than with concrete measurements. For example, where an item about area or perimeter was accompanied by an illustration, students did well. When there was no illustration, they did less well.

Objectives 41 - 44: Algebra

Many students drop out of mathematics after their general mathematics classes, and many school districts allow students to graduate with only one year of high school mathematics which does not have to be algebra. Since few students study algebra, the results are reasonable for the algebra items of the test. For example, an average of 47% of the eleventh grade students evaluated algebraic expressions correctly. In algebra as in other areas, scores were low in subtraction. On the algebra item requiring subtraction, 31% of the students attained the correct answer. If there is reason for concern about the algebra results, that concern should take the form of encouraging students to take more mathematics courses, rather than concern that students did poorly in content which they had not studied formally.

Objectives 45 - 48: Geometry

The geometry section contained interesting results. In the past, it was often said that geometry had not received enough attention. The test results showed that 55% was the average percentage correct by eighth and eleventh graders on the geometry items in the test. Recently the language development approach has been emphasized in the teaching of geometry. If the students tested have been taught according to that approach, the results on the test suggest that it may have been more successful than previous approaches.

Objectives 49 - 52: Probability and Statistics

The results on the probability and statistics items of the test are a pleasant surprise in light of the concern many mathematics educators have had about the limited instruction which students receive in probability and statistics, combinations and permutations, and such areas.

However, one is inclined to think that students are learning these from out-of-school experiences and that they are learning them because the content is of vital interest and because they enjoy the learning. In other words, students are having fun learning what is relevant to them. Research into the circumstances and strategies by which students learn probability outside of school should be considered. Subsequently, schools could initiate strategies which make out-of-school learning an aid to bringing about better in-school learning.

Objectives 53 - 56: Personal and Consumer Mathematics

In the area of applied problems, there was a noticeable relationship between the subject matter of a problem and the scores. The test items focused on problem solving (Objective 56). When an item dealt with what students were interested in, more of them attained the correct answer. For example, students usually are not interested in city property tax, but are interested in swimming. On the item related to swimming pools, 81% of the students were successful, but only 43% calculated property tax accurately. Problems of the difference between total installment cost versus cash price require sophisticated levels of skill. On the test, 58% of the students attained the correct answer for the item on car buying, and 67% were correct in the price differential of refrigerators. Such purchases may be familiar to students. Most eleventh graders want to buy a car whether or not that is a realistic goal for them, and they have some knowledge of the economics involved in such a purchase.

Most of the consumer items involved ratio and proportion. However, the results are inconsistent across the items which indicate that few students saw them as such. If the students had seen them all as problems of ratio and proportion and applied that approach, the results would probably have been more consistent. The concepts and problem-solving strategies accompanying a ratio and proportion approach are tools that students can use to solve many kinds of mathematics problems. In fact, most of the items in the mathematics tests could have been solved by using ratio and proportion. Ratio and proportion is a major problem solving strand not given the curricular emphasis it warrants. It should be a strand through all mathematics courses. If the students understood this strand and how it could be applied, they probably would have done far better than they did on the tests.

Critique

The test is quite good in many ways, and the results provide excellent data for reflection. However, there are some areas that could be improved. Items dealt mainly with material which is taught at the grade levels tested by the tests. The eighth and eleventh grade tests could be improved if they also tested for retention of the mathematics which is taught at lower grade levels. Educators need to know, not only how well skills are learned when first taught, but also how well they are retained years later.

In regard to improvement of items, a more accurate test of students' abilities could be obtained if some of the word problems were rewritten. In particular, some items might be rewritten to correspond with a young person's view of the problem rather than with an adult's view. Sometimes the two views are quite different. Wherever reasonable, items should contain only what is necessary for the solution of the problem. In some items, students seem to have been distracted by the irrelevant information presented in the narrative of the problem. Instead of ignoring that information and working out a problem which they knew how to solve, they became confused by the superfluous information and guessed incorrectly. The test could also be improved by focusing on material that is part of students' everyday experiences and eliminating that which is outside their ordinary realm of interest.

The test could help educators understand more of what children know and do not know if it included concrete representations wherever possible and eliminated most of the problems using abstractions alone. One addition which would be of benefit in future tests would be more items requiring students to use different units of measurement. The present tests contain few items requiring conversions from one measurement unit to another. Finally, there were a few items which did not seem to measure what they were purported to measure. All of the items should be tested and revised as necessary to make future test instruments better.

Conclusion

The major conclusion which emerges from analysis of the tests and results is that they were an excellent beginning. The items covered much of the mathematics taught to average students, and the results provide helpful information. We do not know as much as we would like about the teaching and learning of mathematics, and the results of the 1978 IIEP can stimulate educators to research and rethink our approaches to mathematics' education.

Both in curriculum and in testing, we may do well to consider all of mathematics again and try to identify the general concepts and skills which are most useful to students for solving all kinds of mathematics problems. Analysis of the tests suggests that most of the problems can be solved by an understanding of ratios and proportions, equations, and graphs. Concentration on concepts and manipulations with our "base 10" system would also appear to be generalizable to many problems of mathematics. Learning theorists and researchers generally agree that learning is acquired in less time and retained for a longer time when instruction proceeds from the concrete to the abstract and when multiple experiences with the principles and applications imprint the knowledge deeply in the mind of the learner. Results of the tests make one ask whether these steps are followed in some areas of mathematics. For example, at the fourth grade level, students recognized fractional parts presented in a graphic illustration. However, eighth grade students did not recognize the concept. Also in regard to fractions, intermediate level students are taught addition of fractions with different denominators, yet most eighth and eleventh grade students were not able to add two simple fractions successfully. One cannot be sure whether those who were unsuccessful did not learn the abstract concept in the intermediate grades, did not retain the concept, or both. An additional question is whether they had sufficient experience with fractions. In any case, the situation leads one to the issue of language development. Students often lack the ability to label and understand principles and operations. Perhaps if a language development approach were emphasized, learning would be more effective. Also, regarding fractions, learning might be increased if we began with and frequently repeated concrete representations, helped students to understand that fractions are part to whole relationships, and constantly encouraged them to work with fractions as ratios and proportions.

With the current emphasis on metrics and decimals, there is a trend to suggest curricular de-emphasis of fractions. The results on the tests suggests the contrary. There appears to be a need for more emphasis on fractions.

Students' inability at subtraction is also of major concern. That inability was evidenced at all grade levels and occurred even on an algebra problem. From these results, it is clear that educators would do well to take a good look at subtraction and devote considerable energy to the improvement of student results. Lack of language development was also indicated by student confusion regarding the concepts of area, perimeter, and exponents.

The results on items concerning statistics and graphs should stimulate some interesting speculation. That area has received scant attention in classes attended by average students, yet the student results were at moderate levels on those items. Research regarding the out-of-school learning of students might lead to the discovery of helpful, heretofore unknown, teaching strategies. In regard to in-school learning, students are gaining some understanding of the area of mathematics, but there is considerable room for improvement in almost all areas.

The IIEP tests have been administered and analyzed. The results are published in documents by the Illinois State Board of Education. Mathematics educators throughout Illinois would do well to study the results. The reflections of some mathematics educators are included in this and other publications. These comments can be a stimulus for thought and discussion. The reports contain excellent information. They should not be put on shelves unread. Mathematics educators throughout Illinois should study them, discuss them, and work with the Illinois State Board of Education to help Illinois students learn all that they can about mathematics. Now that the IIEP results are published, educators will want to consider what the next steps should be for future tests, research, curricula, and teaching strategies.

CHAPTER 2

ANALYSIS OF THE FOURTH GRADE RESULTS

What Was Tested

The fourth grade IIEP tested most of the mathematics objectives in Appendix A which are appropriate for fourth graders. It contained items on numeration concepts, properties of numbers and operations, whole numbers, fractions, measurement, geometry, and personal and consumer mathematics. It did not contain items on material which fourth graders have not studied such as decimals, percents, integers, rationals, real numbers, algebra, or probability and statistics.

Summary of Results

The results of the fourth grade test provided some important information. Students did better at addition and multiplication than at subtraction and division. They also indicated a beginning understanding of fractions. Most of the fourth graders seemed to have mastered the basics of telling time. Finally, in the area of measurement, 82% of the students knew the number of inches in a foot, and more students knew basic metric units in 1978 than in 1976.

Although the positive accomplishments of the students should be pointed out, some difficulties must be noted. Students had trouble with subtraction and division. They also had trouble with place value, renaming, and large numbers. For instance, 78% of the students chose the correct response to a division problem with a single digit dividend and divisor, while only 50% were correct on a similar item with a three digit dividend and a single digit divisor. Furthermore, though percentages correct on the 1978 test regarding basic facts concerning measurement were higher than the percentages on the 1976 test, the results were less than most citizens and schools would like. Finally, student ability to analyze and solve problems was low. The transfer of mathematics knowledge and computational skill into an ability to use these learnings to solve mathematical problems successfully continues to be a major challenge to schools.

Objectives 1 - 4: Numeration Concepts

The five items on numeration concepts focused on place value and the ordering of whole numbers. Students did moderately well when the concept was simple, but proficiency was low when the item involved multiple concepts and/or multiple steps.

Objectives 5 - 8: Properties of Numbers and Operations

Most fourth graders attained the correct answers on the items dealing with properties of numbers and operations. Fourth graders seem to be receiving a good introduction to fractions, and most of them seem to grasp the concepts

of odd numbers, greater, lesser, and equal. The three fraction items required recognition of the proportion of shaded to unshaded regions of a given geometric figure. Students who missed the fraction items did not seem to understand that the denominator represents the total number of parts into which the whole is divided, including the numerator. Students probably need more concrete experience with fractional parts.

Objectives 9 - 16: Whole Numbers and Fractions

For straightforward computational items with whole numbers, most of the students attained the correct answer. Students who missed the items probably did so because they had difficulty with regrouping. More students attained the correct answers in addition and multiplication than in subtraction and division. Performing functions (e.g., addition, multiplication) generally is acknowledged to be easier than performing the inverse of those functions (e.g., subtraction, division). One fraction item asked the student how many dots would be left if one-fourth of the dots were taken away. Eight dots were displayed. That item was too difficult since few students have experience with that type of fraction item by fourth grade.

Story problems were more difficult than purely computational items. An average of 62% of the students attained the correct answer for story problems whose solution required addition or multiplication; 28% correctly solved the story problem requiring division; and 35% were correct on the problem requiring two computational steps. Overall, an average of 48% of the students attained the correct answers to these items.

Objectives 37 - 40: Measurement

On items about basic measurement facts, the 1978 fourth grade scores were higher than the 1976 scores. The greatest gains were on the items about units of length, where the student percentages increased by 18 and 19 points. There were two such items. One of the two was about customary units of length, and the other was about metric units. Students did better on customary units than on metric units with 82% answering correctly that there are 12 inches in one foot, and 45% responding correctly that there are 100 centimeters in a meter. Only 43% of the students noted correctly that potatoes are sold by the kilogram and 27% that gasoline is bought by the liter where the metric system is used. One reason for the low scores may be that some programs place little emphasis on the equivalence between metric and customary units of measure. For example, one guideline of many metric education programs is that metric units should be taught in themselves and not compared to customary units.

Fourth grade students have had little exposure to story problems involving measurement, and many of them find such problems difficult. The test results showed the difficulty which students had. On measurement items in the test which required problem solving, the student scores were quite low, and they were lower in 1978 than in 1976.

Measurement was identified in 1977 as an area needing attention. When the 1976 results were publicized in the summer of 1977, the State Board noted that measurement was an area of weakness and urged that special emphasis be given to it. In some aspects of measurement, the scores improved from 1976

to 1978. With continued emphasis, it is likely that there could be even further improvement.

Objecives 45 - 48: Geometry

The only geometry item included in the 1978 test asked the students to indicate which of the figures displayed was a triangle. The same item was in the 1976 test and nearly all the students attained the correct answer in both years.

Objectives 53 - 56: Personal and Consumer Mathematics

On items concerning personal and consumer mathematics, student scores varied greatly. On the item asking the length of time between 4:30 p.m. and 5:00 p.m., 75% of the students attained the correct response. Fifty percent of the students were correct regarding how far a car could go on seven gallons of gasoline if the car got 22 miles per gallon, and 21% of the students answered correctly that four eight-ounce packages of cheese are equal to two pounds. The average percentage correct was nearly identical for the five items in both the 1976 and 1978 test. Results on personal and consumer mathematics items showed that scores were higher on items which were simpler, involved addition or multiplication, and dealt with familiar problem situations. Scores were lower on items which were more complex, involved subtraction or division, and dealt with less familiar situations.

General Implications

Results for the 1978 IIEP do not provide the total picture of elementary mathematics achievement in Illinois. Much more data is needed to arrive at such a picture.

Despite possible criticism of an item here and there, schools and citizens can learn much from the test. Fourth grade students did well in addition, multiplication, telling time, fractions, and the basics of measurement.

At the same time, students did not do well in subtraction, division, use of large numbers, and story problems. Student lack of ability in problem solving may be of particular concern to citizens and schools.

In order to improve future results, particularly in regard to problem solving, one suggestion is to emphasize language development. If students acquire sufficient vocabulary they can understand the labels and concepts of mathematics, which are a necessary set of tools for problem solving. Computational skills are another essential set of tools. Problem solving involves the use of these two sets of tools in proper combination. If language development and computational skills are emphasized, students will have the necessary tools. If the students can then learn to use these tools in proper combination, they will develop good skills in problem solving.

Citizens and teachers have set challenging goals for Illinois students in mathematics. If students are to accomplish these goals, Illinois schools must maintain the levels of success already reached while turning areas of past weakness into areas of future success.

CHAPTER 3

ANALYSIS OF THE EIGHTH GRADE RESULTS

What Was Tested

The eighth grade IIEP measured most of the mathematics objectives in Appendix C which are appropriate to eighth grade students. It contained items on numeration concepts, properties of numbers and operations, whole numbers, fractions, decimals, percents, integers, rational numbers, measurement, algebra, geometry, probability and statistics, and personal and consumer mathematics. It did not contain items on real numbers.

Summary of Results

In general, the mathematics results for the eighth grade were reasonably good regarding computation with whole numbers. The students also did well on computation of integers, that is, with positive and negative whole numbers. They did better on items employing familiar formats than on those using unfamiliar formats and were more successful on items whose solution required one step than on those requiring two or more steps. Comparisons with results for the 1976 and 1978 tests showed that eighth grade scores improved from 1976 to 1978 in basic geometry and that eighth graders scored considerably higher than fourth graders in metrics. Some difficulties also emerged. Students had trouble with exponents. This may have been trouble with the symbol, rather than with the concept. Subtraction and division seemed to be areas of weakness. Scores were low in fractions.

Objectives 1 - 4: Numeration Concepts

Strictly speaking, item #62 (3×10^2) was the only one categorized as testing numeration concepts, and 74% of the students attained the correct response. However, there were two other items which related to exponents; they measured Objective 10. Item #35 (4^3) resulted in 47% correct responses, while item #55 ($9 \times 9 \times 9 \times 9$) returned 71% correct responses. The results on item #62 show that eighth grade students have a beginning understanding of the concept of exponents, and the comparative results for the three items together demonstrate that students have more difficulty with the symbolism than with the application of the concept.

Objectives 5 - 8: Properties of Numbers and Operations

Items testing properties of numbers and operations focused on comparative sizes of decimals, fractional parts of a geometric figure, and the distributive property. The largest decimal was correctly identified by 71% of the students, and 51% correctly chose the smallest decimal. The success rate of eighth graders regarding shaded and unshaded fractional parts of a circle was disappointing. For a fraction item on which 68% of the fourth grade students attained the correct answer, only 60% of the eighth graders chose the correct answer to the same item. The item which tested the distributive property was complex, and the results were inconclusive. Future data is needed to clarify those results.

Objectives 9 - 36: Numbers

Eighth grade scores varied depending on the mathematical processes required and the types of numbers involved. The items which required multiplication and division of whole numbers were answered correctly by 71% of the students. An exception occurred on the exponent item where 47% attained the correct answer. The exponent symbol probably caused difficulty for many students. In dealing with integers (positive and negative whole numbers), 64% of the students added and multiplied correctly, while 38% subtracted and divided successfully. The fact that success rates varied as they did is not unexpected since subtraction and division are inverse functions of addition and multiplication, and performance of the inverse function most frequently is more difficult than performance of the function itself. Results indicated that most students could multiply fractions, order decimals, and convert simple fractions into percents and vice versa. However, only 31% of the students added fractions correctly, and only 28% applied the fraction concept correctly to a real life situation. These same two items were also on the eleventh grade test, and the eleventh grade scores were considerably higher. The higher scores of eleventh graders over eighth graders in fractions may indicate that schools continued to review fraction concepts in high school and that some students who had not mastered them by eighth grade learned them by eleventh grade. The greater success of eighth graders with decimals than with fractions, coupled with the thought that many students may learn fractions best after eighth grade, suggests that a curricular reversal may be worth considering. That is, it may be more effective for students to study decimals before fractions.

Objectives 37 - 40: Measurement

The eighth grade test contained ten items on measurement. On three of four identical metric items, eighth graders scored considerably higher than fourth graders. From that perspective the results are quite positive and encouraging. Since most or all of what students learn about the metric system probably is learned in school, the results indicate that schools are helping students to learn measurement. While keeping this perspective in mind, there is another perspective to consider. The average percentage correct for all measurement items combined, customary units and metrics, was 53% for the eighth grade students. For the five items contained in both the 1976 and 1978 tests, scores remained the same. That is, there were no appreciable differences. In late 1977, the State Board reviewed the results of the 1976 IIEP, identified measurement as an area of weakness, and urged that Illinois schools give greater emphasis to the teaching of measurement. Their analysis and recommendation was made public in the late autumn of the 1977-78 school year and should not have been expected to bring about improvement by February/March of 1978 (just four months later) when the 1978 IIEP was administered. More time is needed to see whether student scores will be improved in the future.

Objectives 41 - 44: Algebra

In regard to the four algebra items included in the test, it should be noted that a minority of students take algebra by eighth grade; the percentages correct reflect that fact. The probability is that the students who answered the items correctly were those who usually do well in mathematics. It is also likely that most of them have been exposed to the concepts tested by the items, informally if not formally.

Objectives 45 - 48: Geometry

The three geometry items dealt with vocabulary and geometric facts. An average of 59% of the eighth grade students correctly identified diameter, right angle, and sphere. For the two items contained in the 1976 and 1978 tests, the 1978 scores were higher than the 1976 scores.

Objectives 49 - 52: Probability and Statistics

Data presented graphically to the students were correctly interpreted and processed by 75% of them. This may indicate that concepts developed through graphic illustrations are learned more readily than those developed through symbolism and narrative.

Objectives 53 - 56: Personal and Consumer Mathematics

The eight items in the category of personal and consumer mathematics required students to solve story problems in a consumer setting. The success rate of these items is proportional to the number of steps and operations needed to solve the problem. Problems requiring one step had a higher success rate than problems requiring two or more steps. The type of operation required to solve the problems also appears to affect the success rate. Problems involving addition and/or multiplication had higher success rates than those requiring subtraction and/or division.

General Implications

In general, students had higher success rates with test items that were presented in a familiar format than with items presented in an unfamiliar format. This indicates that transfer learning is not taking place as much as teachers hope. The remedy for this lack of transfer ability is to teach concepts in as many varied formats as possible. This technique increases the success rate of application of mathematical concepts to new situations, which is a prime goal of mathematics education.

The laboratory approach to learning is a technique that involves students with real objects and situations. It is important that students have opportunities to practice and apply their learning to real objects in the environment. The more opportunities they have, the greater may be their success in learning the concepts involved.

A logical approach based on conceptual hierarchies may not necessarily produce the most effective learning sequence. For example, the "fraction" concept is considered by most teachers to be a prerequisite to the teaching of the decimals. However, the eighth grade results of the 1978 IIEP indicate that student success with decimals is higher than their success rate with fractions. Another example is the logical approach to computation skills which has resulted in a sequence of addition first, subtraction second, multiplication third, and division fourth. The test results show that students have more success with multiplication than subtraction. This finding suggests that a reordering of the sequence should be considered in mathematics curricula.

CHAPTER 4

ANALYSIS OF THE ELEVENTH GRADE RESULTS

What Was Tested

The eleventh grade mathematics section of the IIEP sampled students' knowledge of fractions, ratio and proportion, and elementary probability and statistics. In algebra it tested students' abilities to use algebraic symbolization, manipulate algebra expressions, solve equations and determine values of algebraic expressions and functions. It also tested the students' abilities to compute, measure, and solve percentage problems in a consumer-oriented situation. It did not contain items on decimals, percent, and properties of numbers and operations. It was administered to a random sample of eleventh grade students, regardless of which high school mathematics courses the students had taken.

Summary of Results

The results reflect the responses of students from all ability levels, whether or not they planned to further their education after high school or whether any post-high school education would be college education, technical education, or vocational training.

Summary results indicate that eleventh graders did well on consumer problems when asked to work using percentages or simple ratios and proportions. Also, their scores were surprisingly high on metric items. Eleventh grade scores were higher than eighth grade scores which, in turn, were higher than fourth grade scores. Eleventh grade scores on metric items were also higher in 1978 than in 1976. Scores were reasonably good on items requiring basic algebraic knowledge regarding relationships of numbers and algebraic symbolization. Considering that students have little formal instruction in statistical thinking, they did well on statistical items, where 56% was the average of correct responses.

Student scores were low in areas where they are traditionally low (e.g., algebra, geometry, computations, fractions). A number of students confused the concepts, "area" and "perimeter." Many students had difficulty on computational items and items requiring application to real life situations. An area of particular concern was fractions where only 47% of the eleventh grade students added one-half and one-third correctly. Students had difficulty on algebra items which required them to solve equations, manipulate algebraic expressions, and evaluate expressions and formulas. Only 47% of the students were successful on the basic geometry item in the test.

Objectives 1 - 4: Numeration Concepts

Although the ideas of estimation and rounding are generally not emphasized past elementary school, 62% of the students estimated correctly on the test items. With more practice, a number of others would probably estimate such a problem correctly.

Objectives 9 - 36: Numbers

Computation has historically been the lowest point in standardized mathematical test profiles, ranking consistently under Comprehension and Mathematical Reasoning Ability. This may be a function of the way the standardized achievement tests are designed or the way students respond to those questions. There has been a long standing concern about the quality of computation. Schools have been frustrated in their efforts to solve this problem. Although the low scores in computation will be perceived by some individuals as indicators of a wide-spread deficiency in computation, it should be observed that the average of correct responses on the computational items was 61%.

Objectives 13 - 16: Fractions

Responses for eleventh graders were higher than those of eighth graders in their ability to work with fractions. However, most school districts probably would not be satisfied with a less than 50% achievement level concerning the concept and operations of fractions and would want to devote more curricular attention to achievement in this area.

Objectives 21 - 24: Percent

Eleventh grade students ranged from 45% to 75% on their abilities to use percentages in consumer-oriented mathematical situations. Even though they were better able to apply percentages to real life situations than other mathematical topics, each school district will want to examine whether more stress on consumer-oriented mathematics should be made before eleventh grade.

Objectives 37 - 40: Measurement

It is interesting to note that a surprisingly high percentage of eleventh graders know what the basic units are in the System Internationale metric system, even though they have a lesser knowledge of the common prefixes and the relative sizes of different metric units. It is anticipated that most school districts will feel good about the level of knowledge of the metric system of measurement when contrasted with the amount of this material which has been explicitly taught to date.

Eleventh grade students seem to be able to use ordinary formulas for measurement of area and perimeter approximately as well as they are able to compute, even though there are indications that they confuse the concepts of area and perimeter at times.

Objectives 41 - 44: Algebra

Interpretation of responses to algebra-related items is extremely difficult without knowing how much algebra these eleventh grade students have or have not taken. The students did well on items that reflect general knowledge of relationships of numbers and algebraic symbolization and less well regarding the specifics of solving an equation, manipulating algebraic expressions, and evaluating expressions and formulas for specific numbers.

Objectives 45 - 48: Geometry

There was one geometry item on the test. Only 47% of the students responded correctly that the third angle of a triangle is 25 degrees if the first angle is 50 degrees and the second angle is 105 degrees. Lack of geometric knowledge among average students has been a concern of mathematicians.

Objectives 49 - 52: Probability and Statistics

Since working with chance and probability is a small part of many current mathematics curricula, most school districts would probably be pleasantly surprised at the 56% average of correct responses to the items on probability, statistics, and graphs.

Objectives 53 - 56: Personal and Consumer Mathematics

Eleventh grade students did better on "real life" problems of ratio and proportion than on the classroom mathematics item requiring addition of fractions, but the average of correct responses for the personal and consumer mathematics items was only 56%. School districts may want to give major attention to the use of mathematics for real life situations.

General Implications

The test is a sample of items which measure many of the mathematics objectives in Appendix A. They are made available to local school districts who are responsible for the interpretation of student performance and identification of strengths and weaknesses in their high school mathematics curricula according to their own goals and standards. The focus of the test was on the mathematics of most interest to Illinois citizens. The results are a benchmark of average student performance on the test items across Illinois. The test items are not a random sampling of all mathematics curricular topics, nor are they an adequate reflection of all current mathematical curricular topics. For example, there are no items on different kinds of numbers, axioms or theorems about numbers or geometric figures, trigonometry, or technical mathematics. The multiple choice test format precludes testing more complex mathematical tasks, such as proving theorems, graphing functions, reasoning according to the laws of logic, computer programming, or solving problems involving more than two tasks.

Development of a battery of mathematics tests, which individual school districts would use to assess the state of their students' mathematical knowledge and to compare these findings with the goals of their mathematics curriculum, is a desirable and challenging task. In this development, the selection of individual test items, the relationship of these items to other items in the complete test, and the pattern of student responses to these items are all important in interpreting the results. In other words, one should be cautious about drawing conclusions if the only available information is percentages correct for a few items. In the developmental stages, it is equally important to know whether the incorrect responses were so plausible as to represent minor misunderstanding or so far-fetched as to represent complete lack of knowledge. Students' success at answering a similar question

is also an important consideration. The clarity of wording in the item itself is important in making interpretive conclusions. Total scores of students on the tests may be helpful. The size and nature of the student sample also must be considered in any generalization. The purpose of the test is an essential consideration. If items test mastery of basic material, anything but a high percentage of correct responses is an indictment of students' knowledge and, indirectly, of the effectiveness of our mathematics curricula. But if an item does not represent mastery of fundamentals, a low percentage of correct responses could be interpreted as satisfactory performance.

This publication of results for the IIEP is to be seen as a public sharing of the nature of the 1978 test battery and factual information on student responses. Comments on this information are best regarded as initial observations, rather than conclusions, and curricular implications are best regarded as initial tentative suggestions, rather than firm recommendations.

SECTION II

TEACHER EXPECTATIONS

CHAPTER 5

TEACHER EXPECTATIONS OF STUDENT PERFORMANCE

During the 1978 test, mathematics teachers in sampled schools responded to a survey asking them to estimate the percentage of students who would obtain the correct answer to each mathematics test item. Teachers estimated the performance only of those students whose grade level they taught. Fourth grade teachers estimated performance of fourth grade students on the fourth grade items, eighth grade teachers estimated eighth graders' performance, and eleventh grade teachers estimated eleventh graders' performance. The items were then grouped by objective and compared with actual student performance. Where the same objectives were measured at different grade levels, comparisons were made between teachers' expectations and student performance across grade levels.

Summaries of the teacher expectation data indicated that

- STUDENT SCORES GENERALLY WERE LOWER THAN TEACHER EXPECTATIONS.

- TEACHER EXPECTATIONS WERE ACCURATE REGARDING STUDENT RECOGNITION OF MATHEMATICAL FACTS.

- STUDENT SCORES WERE HIGHER THAN TEACHER EXPECTATIONS REGARDING STUDENT UNDERSTANDING OF CONCEPTS, BUT LOWER THAN EXPECTED REGARDING STUDENT COMPUTATION ABILITY.

- TEACHER EXPECTATIONS WERE MOST ACCURATE REGARDING STUDENT RECOGNITION OF MEASUREMENT FACTS.

- STUDENT SCORES IN PROBABILITY AND STATISTICS SURPASSED TEACHER EXPECTATIONS.

- STUDENT SCORES IN FRACTIONS WERE LOWER THAN TEACHERS HAD ESTIMATED.

- TEACHER ESTIMATES WERE INACCURATE REGARDING STUDENT ABILITY TO DO PROBLEM SOLVING.

Teachers at all grade levels consistently overestimated actual student performance. They overestimated on 10 of the 13 objectives at the fourth grade, on 18 of the 21 objectives at the eighth grade, and on 11 of the 18 at the eleventh grade. Fourth grade teacher estimates averaged 62%; student performance was 56%. Eighth grade teacher estimates averaged 62%; student performance was 52%. Eleventh grade teacher estimates averaged 61%; student performance was 57%.

A total of 31 objectives were measured across the three grade levels. Four objectives were measured at all three grades. Four others were measured at the fourth and eighth grades, eight others at the eighth and eleventh grades, and one other at the fourth and eleventh grades. In addition, four objectives were measured only at the fourth grade, five only at the eighth grade, and five only at the eleventh grade.

Overview of Mathematics Objectives - Implications For Future Research

There appears to be a pattern to the overestimation on the part of teachers toward student achievement across the objectives tested. Teacher expectations were most precise on objectives testing student recognition of mathematical facts. As the skill level being tested became more complex, teacher expectations were more disparate. Teacher expectations were more discrepant on objectives involving computation, understanding, and problem solving.

Student recognition of measurement facts was tested at each grade level. Teacher expectations almost paralleled student performance. The majority of items used to test this objective involved metrics. Fourth grade teachers indicated a lack of exposure to this kind of subject matter, and the results would appear to substantiate that belief. Eighth and eleventh grade teachers reported a larger degree of exposure and student performance was correspondingly higher. Teacher expectations across the three grades were within three percent of actual student performance.

The largest discrepancy in terms of teacher expectations and actual student performance occurred on questions involving computing, understanding, and problem solving with fractions. Eighth and eleventh grade teachers greatly overestimated student ability to compute using fractions. Student understanding of fractions was measured at the fourth and eleventh grades. Fourth grade teachers indicated a lack of exposure to the subject matter and anticipated rather low achievement on the objective. Teacher expectations at both the fourth and eleventh grade were substantially higher than actual student performance. The same was true at the eighth grade where student performance on problem solving was measured. Teacher expectations surpassed actual student performance to a very great extent.

Teacher expectations were consistently lower than actual student achievement on objectives concerning student ability to compute, understand, and solve problems involving probability and statistics. Teacher estimates of student exposure to these mathematical concepts were rather low. In regard to probability and statistics, students are learning more outside of school than teachers think.

Student ability to compute and understand algebra was measured at the eighth and eleventh grade. In spite of the fact that algebra in many schools is an elective and consequently some students may not have been exposed to subject matter of this nature, teachers were fairly accurate in their expectations. Eighth and eleventh grade teachers somewhat overestimated student proficiency in the computation of algebraic problems. They underestimated student ability to understand algebraic problems to the same degree.

CHAPTER 6

RESULTS FOR SPECIFIC OBJECTIVES

Numeration Concepts

Student ability to recognize numeration concepts was measured by Objective 1 (Table 1). At the fourth grade, teachers were very accurate in their estimation of student performance. They stated that the items were of appropriate difficulty level and that students had been exposed to the subject matter. Student performance would appear to reflect that exposure, but students may need additional exposure to numeration concepts in order to retain and understand those concepts.

Objective 2 (Table 1) assessed student ability to compute with numeration concepts at the eighth and eleventh grades. Teachers underestimated student performance at the eighth grade, but overestimated it at the eleventh grade. Teachers at both grade levels indicated that the items were of an appropriate difficulty and that the students had been exposed to the subject matter. Student performance was higher at the eighth grade.

Table 1

Numeration Concepts

Objective	Grade Tested	Teacher Expectation of Student Performance	Actual Student Performance	Underestimation/ Overestimation
Recognition (1)	4	68%	70%	- 2%
Computing (2)	8	65%	74%	- 9%
	11	75%	62%	+13%
Understanding (3)	4	72%	58%	+14%

Student understanding of numeration concepts was measured at the fourth grade by Objective 3 (Table 1). Teachers overestimated student performance. Teachers stated that the items were of an appropriate difficulty level and that students had been exposed to the subject matter. Students were asked which number was greater and were given answer choices where the numerals were written (i.e., four thousand instead of 4,000). In order to answer the question, students had to convert the written number to a numerical one and then respond. Teachers may be assuming students have an adequate background to perform an operation of this nature, but the results do not appear to reflect that background.

Properties of Numbers

Student ability to recognize the properties of numbers was assessed at both the fourth and eighth grades by Objective 5 (Table 2). Fourth grade teacher expectations almost paralleled student performance. Teachers at that grade judged the items used to measure the objective to be of an appropriate difficulty level, but stated that some of the students had not been exposed to the subject matter. Eighth grade teachers overestimated student performance. Even though teacher estimation of student exposure to the subject matter was greater at the eighth grade than at the fourth, actual student performance at the eighth grade was substantially less than teacher expectations. Eighth grade students would appear to have difficulty answering questions involving place value and the distributive properties of numbers.

Eighth grade teachers also overestimated student performance on a similar objective which was assessed only at that grade level. Objective 6 measured student ability to compute with numerical properties. Students were asked to count the number of shaded areas in a circle and reduce that number to a fraction. Teachers were of the opinion that students had been exposed to the subject matter. As can be seen in Table 2, student performance does not coincide with either teacher estimation of student exposure to the subject matter or teacher expectations. But student performance does represent an improvement over eighth grade performance on Objective 5.

Table 2

Properties of Numbers

Objective	Grade Tested	Teacher Expectation of Student Performance	Actual Student Performance	Underestimation/ Overestimation
Recognition (5)	4	68%	69%	- 1%
	8	70%	48%	+22%
Computing (6)	8	75%	60%	+15%
Understanding (7)	4	78%	59%	+14%

Objective 7 (Table 2) measured student understanding of the properties of numbers at the fourth grade. Teachers overestimated student performance. Nine out of ten teachers indicated student exposure to the subject matter. One of the questions used to assess the objective involved subtracting one three digit number from another three digit number, and determining which of the two numbers when added together could be used to check the answer. Student performance would seem to indicate that students need additional instruction in how to check to see if they have obtained a correct answer.

Whole Numbers

The following three objectives assessed student knowledge with respect to whole numbers.

Objective 9, which was only assessed at the fourth grade, measured student ability to recall and recognize whole number facts, definitions, and symbols quickly. Teachers, once again, overestimated student performance. Although teachers stated that students had been exposed to the subject matter, 30% of them reported that the question used to assess the objective was too easy. The item used to measure the objective was a division problem using one digit numbers. As can be seen in Table 3, a large majority of the teachers indicated student exposure to the subject matter. Since the problem used the ÷ symbol, instead of)‾‾, the results may mean that some students have been inadequately exposed to the ÷ symbol.

Student ability to perform computations involving whole numbers was assessed by Objective 10 at all three grades. Teachers at each grade level were consistent in their overestimation of student performance on the objective. Eighth grade teachers slightly overestimated student performance. Fourth and eleventh grade teacher expectations were considerably higher than actual student results. Teachers at each grade level reported a large degree of student exposure to the subject matter. Teachers at the fourth and eighth grades indicated that the questions used to assess the objective were of an appropriate difficulty level. Twenty-five percent of the eleventh grade teachers stated that the questions were too easy.

Table 3

Whole Numbers

Objective	Grade Tested	Teacher Expectation of Student Performance	Actual Student Performance	Underestimation/ Overestimation
Recognition (9)	4	86%	78%	+ 8%
Computing (10)	4	77%	63%	+14%
	8	69%	63%	+ 6%
	11	81%	67%	+14%
Problem Solving (12)	4	58%	47%	+11%
	8	57%	45%	+12%

Objective 12 assessed student ability to solve complex whole number word problems. Teachers at the fourth and eighth grades where the objective was measured, anticipated a rather low performance on the part of students. Their expectations would appear to be supported by student performance. Regardless of the fact that teachers, to a large extent, indicated student exposure to the subject matter, less than one-half of the students at each of the grades responded correctly. Teachers at both grades felt that the questions were of appropriate difficulty. It would appear that if students have not mastered word problems in the lower grades, they will continue to have trouble with them as they progress through higher grades.

Fractions, Decimals, and Percents

Student ability to compute using fractions was measured by Objective 14 (Table 4) at the eighth and eleventh grades. Teachers at both grades were very discrepant in their overestimation of student performance. Once again, teachers overwhelmingly stated that students had been exposed to the subject matter being assessed. However, 34% of the eighth grade teachers and 28% of the eleventh grade teachers indicated that the questions were too easy. Nevertheless, teacher expectations exceeded student performance by 31% at the eighth grade and 24% at the eleventh grade. Approximately one-half of the students at both grades answered correctly. It would appear that very little learning with respect to fractions has occurred between the eighth and eleventh grades. Apparently if a student has not acquired this knowledge by the eighth grade, very few students take additional math courses or have "learned how" between that grade and the eleventh grade.

Objective 15 (Table 4) assessed student ability to understand fractions at the fourth and eleventh grades. Teachers at both grades greatly overestimated student ability. Less than half (48%) of the fourth grade teachers reported student exposure to the subject matter and 27% felt the question was too difficult. Although their expectation of student performance was correspondingly low, student results were 25% below their expectations. Eleventh grade teachers overestimated student performance by 20%. Unlike the fourth grade teachers, they stated that students had been exposed to the subject matter and the question used was of an appropriate degree of difficulty.

Table 4

Fractions, Decimals, and Percents

Objective	Grade Tested	Teacher Expectation of Student Performance	Actual Student Performance	Underestimation/ Overestimation
Computing With Fractions (14)	8	81%	50%	+31%
	11	76%	52%	+24%
Understanding Fractions (15)	4	45%	20%	+25%
	11	66%	46%	+20%
Problem Solving With Fractions (16)	8	60%	28%	+32%
Computing With Decimals (18)	11	55%	64%	- 9%
Computing With Percents (22)	8	59%	48%	+11%

One other objective concerning fractions was measured at the eighth grade. Objective 16 assessed student ability to solve complex fraction word problems. Teacher expectations were substantially greater than student performance. Eighth grade teachers reported a high degree of student exposure to the subject matter, and they felt the difficulty level of the question used to assess the objective was appropriate. Actual student performance, as can be seen in Table 4, was 32% below teacher expectations. The item used to measure the objective asked students to indicate what fractional part of a group were boys, if the group contained 13 boys and 15 girls. Although students in general would appear to have difficulty with math word problems, this difficulty is compounded when the process involves fractions. Students may need additional instruction in the four steps needed to solve word problems.

Objective 18 (Table 4) assessed student ability to compute with decimals at the eleventh grade. Teachers underestimated student performance. Teachers stated that the question used was of an appropriate difficulty and that students had been exposed to the subject matter.

Student ability to compute using percentages was measured by Objective 22 (Table 4) at the eighth grade. Teacher expectations, while rather low, were still higher than actual student performance. Students were asked to determine what 45% of 180 was. Although over 80% of the teachers reported student exposure to the subject matter, less than half of the students were successful on this particular problem. Teachers stated that students had been exposed to the subject matter, and the item difficulty was appropriate.

Integers, Rationals, and Reals

Two objectives assessed student achievement with respect to integers. Objective 26 measured student ability to compute with integers at the eighth grade. As Table 5 shows, student achievement did not meet teacher expectations. Even though nine out of ten teachers reported student exposure to the subject matter and an almost equal number of teachers stated the difficulty of the questions was appropriate, slightly more than half of the students were proficient on the objective. Since operations on positive and negative numbers and exponents are often one of the last chapters in mathematic textbooks, student exposure may very well be brief and may account for the rather low performance on the part of students.

Student ability to solve complex word problems involving integers was measured at the eighth and eleventh grades by Objective 28. Student performance, although not equal to teacher expectations at either grade, did increase 22% from the eighth to the eleventh grades. Teachers indicated a large degree of exposure to the subject matter at both grade levels. They also reported that the question used at both grades to assess the objective was of an appropriate difficulty. Student performance on the previous objective and on this objective at the eighth grade would appear to support a lack of adequate exposure to integers at the eighth grade.

Student achievement on rationals was assessed by two objectives. Objective 29 measured student recognition of rationals at the eighth grade. Teachers overestimated student ability with respect to this objective. They stated students had been exposed to subject matter of this nature, and that the questions used to measure the objective were of an appropriate difficulty.

Table 5

Integers, Rationals, and Reals

Objective	Grade Tested	Teacher Expectation of Student Performance	Actual Student Performance	Underestimation/ Overestimation
Computing With Integers (26)	8	62%	53%	+ 9%
Problem Solving With Integers (28)	8	61%	43%	+18%
	11	73%	65%	+ 8%
Recognition of Rationals (29)	8	66%	57%	+ 9%
Problem Solving With Rationals (32)	8	56%	47%	+ 9%
	11	66%	81%	-15%
Understanding Reals (35)	11	57%	35%	+22%

Student ability to solve complex word problems involving rationals was assessed by Objective 32 at the eighth and eleventh grades. As can be seen in Table 5, slightly more than half of the eighth grade teachers thought students would successfully respond. A student performance of less than 50% confirmed teacher predictions. At the eleventh grade, student performance was greater than teacher expectations. Eighth grade teachers stated that the question was of an appropriate difficulty, but many students had not been exposed to the subject matter. Eleventh grade teachers indicated that the question was appropriate in difficulty, and students had been exposed to the subject matter. Student performance at the eleventh grade represents a 34% improvement from that of the eighth grade. The increase may be a function of the degree of exposure to rationals between the eighth and eleventh grades, as the teachers would appear to indicate.

Objective 35 (Table 5) measured student understanding of reals at the eleventh grade. Eleventh grade teacher predictions of a low student achievement on this objective were sustained by an even lower student performance. Teachers stated that the question was of an appropriate difficulty, and that students had been exposed to the subject matter. The problem asked for students to indicate which number was the closest to the square root of ten. Students who had not taken a math class for several years may have had difficulty in recalling this concept. If this were the case, it could account for the rather low performance.

Measurement Concepts

Four different objectives were used to assess student achievement on measurement problems. Objective 37 assessed student ability to recognize measurement facts, definitions, and symbols quickly. The objective was measured at each grade level. As Table 6 indicates, teacher expectations were very accurate with respect to actual student performance.

Eighth and eleventh grade teachers indicated that the questions used to assess the objective were of an appropriate degree of difficulty, and students had been exposed to the subject matter. Less than half of the fourth grade teachers thought that students had been exposed to the subject matter, and a large number of them felt the questions used were too difficult. The majority of the questions used to assess the objective involved metrics. Fourth grade teacher expectations of student exposure to subject matter of this nature and their speculation that the questions were too difficult would appear to be confirmed. Student performance at the fourth grade was less than 50%.

Table 6

Measurement

Objective	Grade Tested	Teacher Expectation of Student Performance	Actual Student Performance	Underestimation/ Overestimation
Recognition (37)	4	45%	44%	+ 1%
	8	65%	64%	+ 1%
	11	66%	69%	- 3%
Computing (38)	8	55%	36%	+19%
	11	62%	50%	+12%
Understanding (39)	4	52%	32%	+20%
	8	58%	40%	+18%
Problem Solving (40)	4	61%	48%	+13%
	8	63%	51%	+12%
	11	57%	48%	+ 9%

Objective 38 (Table 6) assessed student ability to perform measurement computations and was assessed at the eighth and eleventh grades. Although teachers at both grades anticipated a rather low student result, they overestimated student performance. Teachers at both grades indicated students had been exposed to the subject matter, and the questions used to assess the objective were of an appropriate difficulty, but student performance did not exceed 50% at either grade.

Student ability to understand measurement concepts, facts, and processes was assessed by Objective 39 (Table 6) at the fourth and eighth grades. Teacher expectations at both grades, although low, substantially overestimated student performance. At the fourth grade, approximately half of the teachers indicated students would correctly respond to an item which asked how many ounces there were in two pounds. Their expectations exceeded student performance by 29%. Fourth grade teachers indicated that the question was of appropriate difficulty, but expressed some reservations as to whether or not students had been exposed to the subject matter. Students at the eighth grade were given a rectangle and asked to indicate which formula measured the distance around it. Eighth grade teachers were similar to fourth grade teachers in their degree of overestimation (18%). Eighth grade teachers stated that students had been exposed to the subject matter, and the questions used were of an appropriate difficulty.

One other objective involving measurement was assessed at the three grade levels. Objective 40 (Table 6) tested student ability to solve complex measurement word problems. Teacher expectations at all three grades were rather low. Regardless of this, teachers overestimated student performance at each grade. Sixteen percent of the fourth grade teachers were of the opinion that the questions were too difficult. There would also appear to be some disagreement as to whether fourth graders had been exposed to the subject matter. Eighth and eleventh grade teachers felt students had been exposed to the subject matter, and the questions measuring the objective were of an appropriate degree of difficulty. Actual student performance at those two grades neither reflects teacher estimation of student exposure to the subject matter, nor their expectations of student success.

Algebra and Geometry

Two objectives concerning student achievement with respect to algebra were measured at the eighth and eleventh grades.

Objective 42 (Table 7) assessed student ability to perform algebraic computations. Teachers at both the eighth and eleventh grades overestimated student performance. Teacher expectations of a rather low performance (both were in the 50th percentile) appear to be reflected in the results. Eighth grade teachers stated that although the questions were of an appropriate level of difficulty, some students may not have been exposed to the subject matter. The results would appear to support that reservation. Eleventh grade teachers indicated that students had been exposed to the subject matter, and the questions used to measure the objective were of an appropriate degree of difficulty. On the whole, student performance does not reflect teacher estimation of student exposure to the subject matter at either grade.

Student understanding of algebra was measured at the eighth and eleventh grades by Objective 43 (Table 7). Teacher expectations underestimated student performance at both grades. Eighth grade teachers were of the opinion that students had not been exposed to the subject matter and questioned the difficulty level of the questions used. Fifty percent reported student exposure and over 25% thought the questions were too difficult. Eleventh grade teachers reported that students had been exposed to the subject matter, and the questions used to assess the objective were judged to be appropriate. Actual student results would appear to support eighth grade teacher estimates of student exposure to this concept. Apparently some learning or additional math learning in algebra has occurred during the interval between the eighth and eleventh grades, accounting for the increase in achievement.

Table 7

Algebra and Geometry

Objective	Grade Tested	Teacher Expectation of Student Performance	Actual Student Performance	Underestimation/ Overestimation
Computing in Algebra (42)	8	53%	42%	+11%
	11	59%	51%	+ 8%
Understanding Algebra (43)	8	40%	50%	-10%
	11	56%	64%	- 8%
Recognition of Geometry (45)	4	85%	95%	-10%
	8	63%	59%	+ 4%
Understanding Geometry (47)	11	63%	47%	+16%

Objective 45 (Table 7) assessed student ability to recognize geometry facts at the fourth and eighth grades. Fourth grade teachers underestimated student performance, and at the eighth grade teacher expectations were close to actual student results. The question measuring the objective at the fourth grade asked students to indicate which figure was a triangle. Fourth grade teachers indicated student exposure to subject matter of this nature, and also stated the question was of an appropriate degree of difficulty. Student performance confirms teacher estimates of exposure. Eighth grade student results were lower than fourth grade student performance. The reason for this discrepancy is that the questions used to measure the objective were of a greater degree of difficulty. For example, one of the questions asked students to identify which line segment was the diameter of a circle. Eighth grade teachers felt the questions were of an appropriate degree of difficulty, but questioned whether all students had been exposed to the subject matter. Notwithstanding this, eighth grade teacher expectations were accurate with respect to actual student performance.

Objective 47 (Table 7) assessed student understanding of geometric concepts at the eleventh grade. Teachers overestimated student performance. Less than half of the students when given the degree of two angles of a triangle were able to determine the degree of the third angle. Although teachers indicated that students had been exposed to this concept, it is quite possible that some students had not been exposed to it. The results may be indicative of students who were not exposed to the subject matter in the eighth grade and did not take math courses where this concept would be taught in high school. Eleventh grade teachers felt that the questions used to measure the objective were of an appropriate degree of difficulty.

Probability and Statistics and Consumer Math

Student achievement with respect to probability and statistics, specifically performing computations, was assessed by Objective 50 (Table 8). At the eleventh grade, teachers underestimated student performance. Less than half of the teachers thought students would be successful on these types of mathematical problems. A large number of teachers indicated that students had not been exposed to this type of subject matter. Student performance closely paralleled teacher estimates of exposure (66%). Teachers were of the opinion that the questions were appropriate in difficulty level.

Objective 51 (Table 8) measured student ability to understand probability and statistical concepts at the eighth and eleventh grades. Teachers underestimated student performance at both grade levels. At the eighth grade, where the question involved reading a graph, three-fourths of the students demonstrated this ability. Eighth grade teachers stated that the question was of an appropriate difficulty, but that many students had not been exposed to the subject matter. Less than 40% of the eleventh grade teachers indicated that students had been exposed to subject matter involving the understanding of statistical concepts. Slightly better than half of the students responded correctly to the questions measuring this objective. One of the questions involved the probability of heads on a coin turning up after four tosses. Teachers stated that they had some reservations as to the level of difficulty of the questions measuring this objective. The lower level of student achievement at the eleventh grade may be a consequence of lack of student exposure to the subject matter or the difficulty level of the questions used at the eleventh grade.

Table 8

Probability and Statistics and Consumer Math

Objective	Grade Tested	Teacher Expectation of Student Performance	Actual Student Performance	Underestimation/ Overestimation
Computing Probability and Statistics (50)	11	49%	62%	-13%
Understanding Probability and Statistics (51)	8	61%	75%	-14%
	11	37%	54%	-17%
Problem Solving Probability and Statistics (52)	11	36%	50%	-14%
Problem Solving Consumer Math (56)	4	61%	49%	+12%
	8	64%	60%	+ 4%
	11	60%	56%	+ 4%

Student ability to solve complex probability and statistical word problems was measured by Objective 52 (Table 8) at the eleventh grade. Teachers underestimated student performance. As they did on the previous two objectives, teachers reported a low level of student exposure to subject matter involving probability and statistics. As a result, their expectations of student performance were correspondingly low. In the questions used to assess the objective, students were told that if in a game they had a ten percent chance of scoring 20 points, at that percentage, in the long run how many points would they score per game. Even though student performance was greater than teachers anticipated, the rather low performance on the part of students may be a reflection of a lack of exposure to this type of skill. Over 25% of the teachers also felt that the question used to measure the objective was too difficult.

The last objective assessed in the mathematics portion of the 1978 test involved student ability to solve complex personal and consumer mathematics word problems. Objective 56 was measured at the fourth, eighth, and eleventh grades. Fourth grade teachers overestimated student performance. Eighth and eleventh grade teachers were accurate in their estimation of student performance. As can be seen in Table 8, student performance increased from the fourth to the eighth grade and decreased from the eighth to the eleventh grade. Exposure to this type of subject matter was reported at all three grades. The questions measuring the objective were also regarded by teachers as being of an appropriate degree of difficulty. It would appear, by the performance of students from the fourth to the eleventh grade, that students either learn to do story problems or they never do.

APPENDIX A

PROCESS USED TO OBTAIN THE CURRICULAR ANALYSES

Mathematics educators from every part of Illinois were nominated by mathematics curricular experts contacted from the Illinois State Board of Education. Nine of those nominated were selected to represent teachers, administrators, and curriculum experts. An effort was made to represent small, medium, and large schools and districts as well as the different geographical areas of Illinois.

The nine reviewers received copies of the test booklets and the student scores for each item. They wrote their analysis and interpretation of the student results for each item.

A second group of reviewers was chosen to critique and edit the interpretations of the first reviewers.

The second group of reviewers analyzed the results by grade level and drafted the grade level curricular analyses. A mathematics expert was selected from outside the group to critique all previous analyses and draft the overview curricular analysis paper.

The papers were reviewed, critiqued, and edited numerous times by mathematics and testing experts inside and outside the Illinois State Board of Education.

APPENDIX B

DESCRIPTION OF THE TEACHER EXPECTATION RESEARCH

At the time the 1978 IIEP was administered, principals of participating schools were asked to select two teachers familiar with the mathematics being taught at the appropriate grade level. Before seeing any student results, the teachers examined each item in the test. They stated whether the students had been exposed to the subject matter and whether the item was of appropriate difficulty level. They also estimated the percentage of students who would obtain the correct answer.

The teacher responses were analyzed and the results are reported in Section II of this report.

APPENDIX C

THE MATHEMATICS OBJECTIVES OF THE IIEP AND THEIR DEVELOPMENT

The IIEP is an objective referenced test. A committee of Illinois experts developed the mathematics objectives.

Mathematics educators were nominated by peers. The nominees were reviewed by consultants and Illinois State Board of Education staff; six persons were selected for the committee. The committee was representative of Illinois geographical areas and types of districts. It included mathematics educators with elementary, junior high, high school, and college teaching experience, and administrative experience.

The committee members and Illinois State Board of Education staff collected sets of objectives. Subsequently, the committee spent two days drafting and refining objectives for the IIEP. During the two-day period, the committee members pilot tested the objectives on a number of test items, refining the objectives after each pilot test. They returned home for a period of two months to reflect on the objectives and pilot tested the objectives on additional test items.

Finally, the committee reconvened, refined the objectives, and pilot tested them on a third set of test items. The committee reached consensus on the mathematics objectives for the IIEP. Presentation and description of the IIEP mathematics objectives follows.

IIEP MATHEMATICS OBJECTIVES

The conceptual foundation of the mathematics objectives of the IIEP is the set of mathematics categories and abilities described below. The categories are mathematics topics. Some categories are self-explanatory; others are less familiar. The self-evident categories are listed without further description; the less familiar categories are described. A list of mathematics abilities follows the list of mathematics categories. The abilities are skills required for success in mathematics. Each mathematics objective describes a particular ability in reference to a specific category.

Mathematics Categories

I. NUMERATION CONCEPTS. *This category refers to the concepts of numeration and place value and the processes of naming numerals, approximating numbers, and rounding off numbers.*

II. PROPERTIES OF NUMBERS AND OPERATIONS. *This category also includes characteristics of numbers and operations and comparisons among numbers.*

III. NUMBERS.

- A. WHOLE NUMBERS. *Whole numbers are the numbers used by children to count. Whole numbers include 0, 1, 2, 3, etc.*
- B. FRACTIONS.
- C. DECIMALS.
- D. PERCENT.
- E. INTEGERS. *Integers are positive and negative whole numbers and zero as distinguished from fractions. The numbers -3, -2, -1, 0, +1, +2, +3, etc. are integers.*
- F. RATIONALS. *Rationals is an all inclusive term for categories A through E, both positive and negative. Examples are +2, +½, +.50, +50%, -2, -½, -.50, and -50%.*
- G. REALS. *Reals is an all inclusive term for categories A through F and numbers such as* π, $\sqrt{2}$, *etc.*

IV. MEASUREMENT.

V. ALGEBRA.

VI. GEOMETRY.

VII. PROBABILITY AND STATISTICS.

VIII. PERSONAL AND CONSUMER MATHEMATICS.

Mathematics Abilities

1. Ability to recall and recognize facts, definitions, and symbols quickly. *Perception is the primary mental act used.*

2. Ability to perform computations, procedures, and complex counting where the operations are indicated.

3. Ability to understand concepts, facts, and processes. *The mental operations of analysis and synthesis are used to make comparisons and evaluative judgments.*

4. Ability to solve complex word problems. *Several of the following operations must be involved: interpretation of the question; identification of the relevant data from the given information; decisions about which operations need to be performed on the data; correct performance of the operations; interpretations of the results.*

Mathematics problems can be placed into one of the matematics categories above. The solution to the problem involves one of the four abilities listed. The matrix of mathematics categories and abilities on the following page is a graphic display of the conceptual model upon which the mathematics portion of the IIEP rests. The mathematics objectives are the cells of the matrix.

Mathematics Objectives

The mathematics objectives of the IIEP are described in the matrix below. This mathematics matrix shows that there are 56 objectives which could be tested by the IIEP.

MATRIX OF MATHEMATICS OBJECTIVES

MATHEMATICS CATEGORIES BY ABILITIES

Mathematics Categories	Mathematics Abilities 1 Ability to recall and recognize facts, definitions, and symbols quickly.	2 Ability to perform computations, procedures, and complex counting where the operations are indicated.	3 Ability to understand concepts, facts, and processes.	4 Ability to solve complex word problems.
I. NUMERATION CONCEPTS	(1) *4th	(2) 8th, 11th	(3) 4th	
II. PROPERTIES OF NUMBERS AND OPERATIONS	(5) 4th, 8th	(6) 8th	(7) 4th	
III. NUMBERS				
A. WHOLE NUMBERS	(9) 4th	(10) 4th, 8th, 11th		(12) 4th, 8th
B. FRACTIONS		(14) 8th, 11th	(15) 4th, 11th	(16) 8th
C. DECIMALS		(18) 11th		
D. PERCENT		(22) 8th		
E. INTEGERS		(26) 8th		(28) 8th, 11th
F. RATIONALS				(32) 8th, 11th
G. REALS			(35) 11th	
IV. MEASUREMENT	(37) 4th, 8th, 11th	(38) 8th, 11th	(39) 4th, 8th	(40) 4th, 8th, 11th
V. ALGEBRA		(42) 8th, 11th	(43) 8th, 11th	
VI. GEOMETRY	(45) 4th, 8th		(47) 11th	
VII. PROBABILITY AND STATISTICS		(50) 11th	(51) 8th, 11th	(52) 11th
VIII. PERSONAL AND CONSUMER MATHEMATICS				(56) 4th, 8th, 11th

Grade levels where objectives were tested in 1978 are not shaded.

APPENDIX D

INDEX OF MATHEMATICS OBJECTIVES AND ITEMS FOR THE 1978 IIEP

FOURTH GRADE		EIGHTH GRADE		ELEVENTH GRADE	
Objective	Item	Objective	Item	Objective	Item
1	27	2	62	2	55
1	28	5	39	10	29
1	47	5	40	10	53
3	48	5	53	14	30
3	49	6	52	14	31
5	29	10	35	14	54
5	37	10	36	15	41
5	38	10	55	18	60
5	39	12	57	28	56
5	53	14	29	32	39
7	35	14	30	35	59
7	50	16	43	37	49
9	26	22	28	37	50
10	30	26	31	37	51
10	31	26	32	37	64
10	32	26	33	37	75
10	42	26	34	38	38
10	51	26	54	38	73
12	41	28	56	38	76
12	54	29	50	38	77
12	55	29	51	40	46
12	56	32	61	40	58
12	59	37	48	42	32
12	60	37	49	42	33
15	52	37	60	42	34
37	33	37	70	42	35
37	46	37	71	42	36
37	61	38	72	42	37
37	62	38	73	42	61
37	63	39	64	42	62
37	64	40	44	42	63
37	65	40	46	42	66
39	34	42	37	43	52
40	36	42	38	43	65
40	43	43	63	47	71
40	45	43	65	50	68
40	57	45	41	50	72
40	58	45	42	51	67
45	40	45	67	51	69
56	36	51	66	51	70
56	41	56	44	52	40
56	43	56	45	56	42
56	44	56	46	56	43
56	45	56	47	56	44
56	57	56	58	56	45
56	58	56	59	56	47
56	59	56	68	56	48
		56	69	56	57
				56	58
				56	74

APPENDIX E

LIST OF PUBLICATIONS DESCRIBING THE 1978 IIEP RESULTS

Executive Summary

Mathematics Curricular Analyses and Teacher Survey Results

Reading Curricular Analyses and Teacher Survey Results

Citizenship Curricular Analyses and Teacher Survey Results

Mathematics Item Results

Reading Item Results

Citizenship Item Results

Fourth Grade Status Report

Eighth Grade Status Report

Eleventh Grade Status Report

These publications are available upon request.

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